

A proposed modification of the dynamic subgrid scale closure method

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Napoli, 24-2-2004.

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A popular choice for modelling turbulence subgrid tensor τ_{ij} consists of introducing the concept of “eddy viscosity” ν_{turb} :

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = -2\nu_{turb}\bar{S}_{ij} \quad (1)$$

where \bar{S}_{ij} is the shear stress filtered on the scale Δ . The quantity ν_{turb} characterizes the turbulence. Its order of magnitude is given by the product of a length times a velocity characteristic of the turbulent motion: $\nu_{turb} \sim l \cdot V_l$. Choosing the length equal to the computational grid, $l = \Delta$, and $V_\Delta = |\bar{S}| \Delta$, we obtain the Smagorinsky model [1]:

$$\nu_{turb} = C\Delta^2 |\bar{S}| \quad (2)$$

The quantity C can be calculated dynamically from the Germano identity:

$$\tau_{ij}^{2\Delta} - \widehat{\tau_{ij}^\Delta} = L_{ij} \quad (3)$$

which furnishes:

$$-2\Delta^2 C = \frac{\langle L_{ij} \widehat{\bar{S}_{ij}} \rangle}{\langle \widehat{\bar{S}_{ij}} \bar{S}_{ij} \rangle} \quad (4)$$

The behaviour of the parameterization (4) is very similar to the dynamic Smagorinsky model formulated in [3]. Hence the tensor given by equation (4) is:

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = \frac{\langle L_{ij} \widehat{\bar{S}_{ij}} \rangle}{\langle \widehat{\bar{S}_{ij}} \bar{S}_{ij} \rangle} \bar{S}_{ij} \quad (5)$$

In order to eliminate the constrain that τ_{ij} is parallel to \bar{S}_{ij} , we propose the following parameterization:

$$\tau_{ij} = -2\Delta^{4/3}(\bar{S}C_{ij} + c\bar{S}_{ij}) \quad (6)$$

where \bar{S} is a quantity representative of the strain-rate (for instance one invariant of the tensor \bar{S}_{ij}). The tensor components C_{ij} are calculated from Germano's identity directly, without introducing the concept of eddy viscosity, whereas the scalar quantity c is the one that better reproduce the dissipation energy.

Combining (6) and (3) we obtain:

$$-2(2\Delta)^{4/3}(\widehat{\bar{S}C_{ij}^{(2\Delta)}} + c\widehat{\bar{S}_{ij}}) + 2\Delta^{4/3}(\bar{S}\widehat{C_{ij}} + c\widehat{\bar{S}_{ij}}) = L_{ij} \quad (7)$$

Assuming that $C_{ij}^{(2\Delta)} = C_{ij}^{(\Delta)}$ and $\widehat{\bar{S}C_{ij}} \simeq \widehat{\bar{S}}C_{ij}$, we have:

$$-2\Delta^{4/3}(\widehat{\bar{S}C_{ij}^{(2\Delta)}} + c\widehat{\bar{S}_{ij}}) = aL_{ij} \quad (8)$$

$$-2\Delta^{4/3}C_{ij} = a\frac{L_{ij}}{\widehat{S}} + 2\Delta^{4/3}c\frac{\widehat{S}_{ij}}{\widehat{S}} \quad (9)$$

In order to calculate c we multiply eq.(8) by \widehat{S}_{ij} and assume that the following relationship is valid for the average quantities:

$$\langle \widehat{S}C_{ij}\widehat{S}_{ij} \rangle \simeq c\langle \widehat{S}_{ij}\widehat{S}_{ij} \rangle \quad (10)$$

Using the above relationship we have:

$$2\Delta^{4/3}c = \frac{a}{2} \frac{\langle L_{ij}\widehat{S}_{ij} \rangle}{\langle \widehat{S}_{ij}\widehat{S}_{ij} \rangle} \quad (11)$$

Hence the tensor τ_{ij} becomes:

$$\tau_{ij} = a\frac{\overline{S}}{\widehat{S}}L_{ij} + \frac{a}{2} \frac{\langle L_{ij}\widehat{S}_{ij} \rangle}{\langle \widehat{S}_{ij}\widehat{S}_{ij} \rangle} \left(\overline{S}_{ij} - \frac{\overline{S}}{\widehat{S}}\widehat{S}_{ij} \right) \quad (12)$$

and assuming $\overline{S} = \sqrt{\overline{S}_{ij}\overline{S}_{ij}}$ we obtain the final form:

$$\tau_{ij} = a\frac{\sqrt{\overline{S}_{ij}\overline{S}_{ij}}}{\sqrt{\widehat{S}_{ij}\widehat{S}_{ij}}}L_{ij} + \frac{a}{2} \frac{\langle L_{ij}\widehat{S}_{ij} \rangle}{\langle \widehat{S}_{ij}\widehat{S}_{ij} \rangle} \left(\overline{S}_{ij} - \frac{\sqrt{\overline{S}_{ij}\overline{S}_{ij}}}{\sqrt{\widehat{S}_{ij}\widehat{S}_{ij}}}\widehat{S}_{ij} \right) \quad (13)$$

We can see that the tensor (13) contains the first term that is similar to the tensor predicted by similarity models [5]:

$$\tau_{ij} = C_{sim}L_{ij} \quad (14)$$

and a second term similar to the dynamic Smagorinsky models such as (5). However, in the case of (13), the term proportional to L_{ij} derives from the Germano identity directly. It is well-known from the literature, that similarity models better reproduce the SG tensor components, whereas models based on the concept of eddy viscosity (1), such as dynamic Smagorinsky model, better reproduce dissipated energy [8]. Tensor (13) may be able to better reproduce both SG tensor components and dissipated energy.

Acknowledgments

M.V. Salvetti is acknowledged for her comments and suggestions.

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